Pitch Sets

Set theory defined

Set theory is the study of pitch in a musical composition by dividing it into meaningful groups of notes that have a distinctive intervallic relationship. This study involves the observation of intervallic groups in melody (related to the study of *motive*) and harmonically (related to the study of *harmony*).

All music can be considered from a standpoint of set theory. For example, a set-theory approach to common-practice music would observe the prevalence of two intervals—the major third and minor third—as dominant constructive elements in both harmony (through the use of triads) and melodically (through melodic arpeggiation).

In set theory, pitches and intervals are both often referred to as **chromatic integers**. Such a system typically uses C as a "fixed DO," so the notes C, C♯ and D are referred to as 0, 1 and 2, and so forth. The intervallic distance between two notes is measured in half steps: a unison is interval 0, a half-step is interval 1, a whole step interval 2, and so forth.

By this logic, a C minor triad would have the pitches 0, 3 and 7; its intervals would be 3 (from C to E♭), 4 (from E♭ to G), 7 (from C to G) and 5 (from G to an upper octave of C).

Organizing pitch sets

Normal order is the most "compact" form of a group of pitches. How to find a set's **normal order:**

1. Write all pitches in ascending order,
2. Find the smallest possible **outside interval** (first to last notes), and
3. If there are multiple forms with the same outside interval, the normal form has the smallest first interval, or is the most "tightly packed to the left."

The inversion of a pitch set is often musically important, and is frequently used as in music that uses pitch sets as a constructive element. How to find a set's **inversion:**

1. Keep the first and last notes the same, and
2. Reverse the interval pattern.

Best normal order compares a pitch set against its inversion. This is particularly useful for grouping a set with its inversion under a single name. By expressing that two sets are "equivalent," they can be discussed in musical analysis as a single entity. How to find **best normal order:**

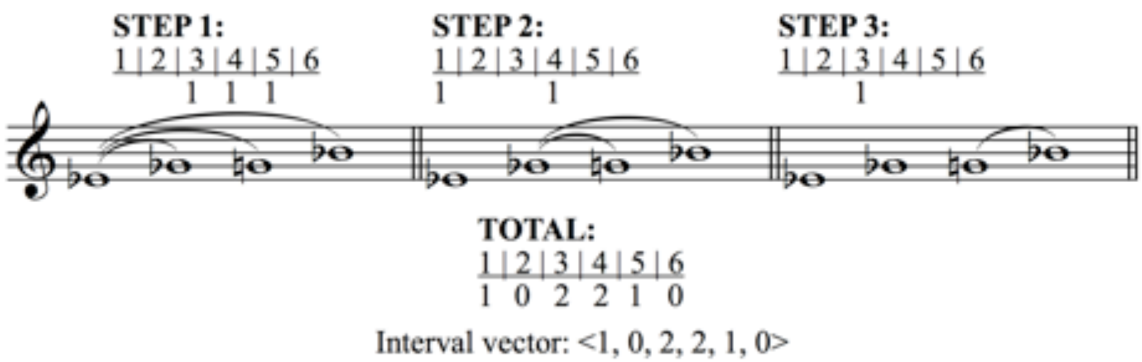
1. Compare the set's normal order with its inversion – the one that is "best" according to "normal order" rule #3, above is the "best normal order."

Prime form is the expression of a pitch set beginning on zero. By using prime form, a pitch set can be expressed under a single name (like saying "minor triad" to express the similarity between any minor triad built on any pitch). How to find a set's **prime form:**

1. Label the first pitch of its *best normal order* “0”
2. Calculate the distance in semitones to each member of the set.

Interval vector is a tool for understanding the interval “content” of a pitch-set. This helps to describe what makes a particular set have a distinctive quality. For instance, the prevalence of thirds and sixths gives a set a colorful quality that is lacked in sets that only contain fourths and fifths. An *interval class* includes an interval and its complement; for instance, interval 3 (minor third) has a complement of 9. How to find a set’s **interval vector:**

1. Write the set in normal order,
2. Make a table of *interval-classes,* like so: 1 | 2 | 3 | 4 | 5 | 6
3. Note number of occurrences of each interval-class in the set by tabulating the intervals from the first note to each additional note, then the second to each additional note, etc.



Interval class **1**: 1 and 11 (m2 & M7)
Interval class **2**: 2 and 10 (M2 & m7)
Interval class **3**: 3 and 9 (m3 & M6)
Interval class **4**: 4 and 8 (M3 & m6)
Interval class **5**: 5 and 7 (P4 & P5)
Interval class **6**: 6 (tritone)

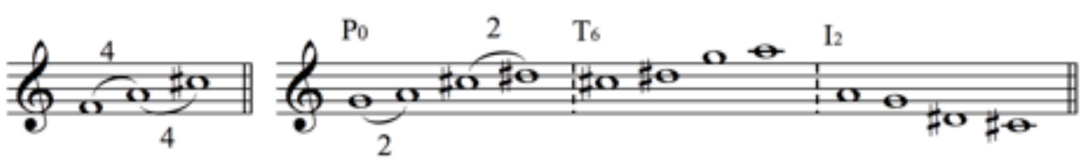
How pitch sets are used

Pitch sets may also be **subsets** of larger sets or common modes. For instance, the set (0, 1, 2) is a subset of the chromatic mode, and (0, 2, 4) is a subset of the chromatic, diatonic and whole-tone modes.

Pitch sets may be related to each other by **common tones**. An example from tonal music is closely related keys: any key shares six notes in common with its dominant key.

Pitch sets may be related by **invariance** under inversion or transposition. This means that individual notes of a pitch-set remain despite the transformation.

Inversionally-symmetrical sets are strongly invariant under inversion or transposition.



Pitch sets can be related by having the same **interval content.** The sets (0, 1, 4, 6) and (0, 1, 3, 7) have the same interval content: <1,1,1,1,1,1>. Theorist Allen Forte called these "**z-related**" sets (the "z" doesn't stand for anything in particular): distinct set-classes with the same interval content. These set-classes are, respectively, 4-Z15 and 4-Z29.

Because sets can function as subsets, a set may have **tonal implications** that can be used to advantage in atonal music.

Berg: *Schließe mir die Augen beide*